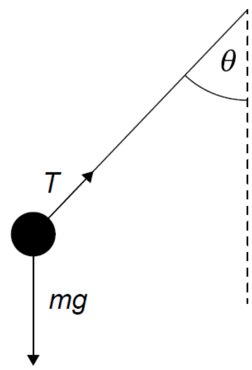
Teacher notes
Topic A

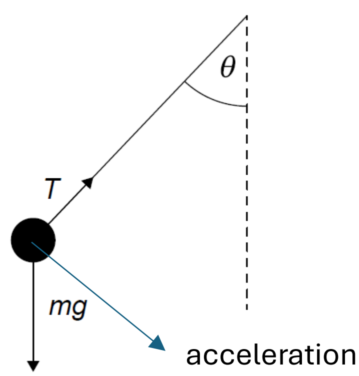
Another question from the M2026 exam.

The mass at the end of the string is released from rest. What is the tension in the string at this instant?



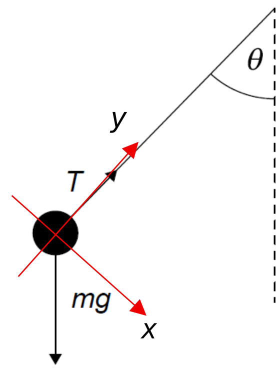
The easiest way to get the answer is to analyze forces and apply Newton's second law. But which axes should we take? The answer is that **any** set of axes will do but some axes are more convenient than others. **The wise choice is to choose the direction of acceleration to be along one of the axes**.

In this case, the ball will accelerate at right angles to the string:



Therefore, the wise choice of axes is:

:



The magnitudes of the components of T and of the weight are:

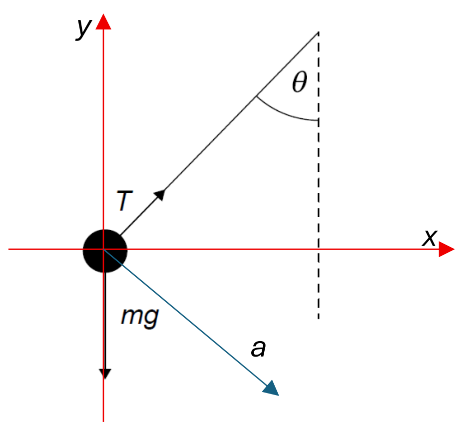
$T_x = 0$ and $T_y = T$

$W_x = mg\sin\theta$ and $W_y = mg\cos\theta$

The net force in the x-direction is $W_x = mg\sin\theta$, the tangential acceleration a and so $ma = mg\sin\theta$ giving $a = g\sin\theta$.

In the y-direction the net force is $T - W_y = T - mg\cos\theta$ and so $T - mg\cos\theta = m\frac{v^2}{L}$. But $v = 0$ so $T = mg\cos\theta$.

What if we chose horizontal and vertical axes?



The magnitudes of the components are now:

$T_x = T\sin\theta$ and $T_y = T\cos\theta$

$W_x = 0$ and $W_y = mg$

The acceleration *a*, is normal to the string, so it makes an angle θ to the horizontal. The acceleration components are:

$a_x = a\cos\theta$ and $a_y = a\sin\theta$

Newton's second law says

$T\sin\theta = ma_x$ and $mg - T\cos\theta = ma_y$

In other words

$$T\sin\theta = ma\cos\theta \qquad (1)$$

and

$$mg - T\cos\theta = ma\sin\theta \qquad (2)$$

The common mistake with this choice of axes is to forget that we have an acceleration component in the *y*-direction and write **incorrectly** that

$$mg - T\cos\theta = 0$$

$$T = \frac{mg}{\cos\theta}$$

From (1) $ma = \dfrac{T\sin\theta}{\cos\theta}$ and substituting this in (2) we get

$$mg - T\cos\theta = \frac{T\sin\theta}{\cos\theta}\sin\theta$$

$$mg = T\cos\theta + \frac{T\sin\theta}{\cos\theta}\sin\theta$$

$$mg = \frac{T\cos^2\theta + T\sin^2\theta}{\cos\theta}$$

$$mg = \frac{T}{\cos\theta}$$

$$T = mg\cos\theta$$

as before.

Clearly the first choice of axes is the better one: it is much shorter and avoids possibilities for errors.